Convex Hull Convolutive NMF for Uncovering Temporal Patterns in Multivariate Time-Series Data

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Motivation
- Time-series data often contains rich structure
- Goal: automatically extract temporal patterns from a time-series

NMF Background
- Non-negative Matrix Factorization: decompose non-negative matrix \( V \) into product of two non-negative matrices \( WH \)

$$V \approx WH$$

- Convolutive NMF: consider temporal context in the input and learn a time-varying dictionary

$$V \approx W(t) \text{ time-varying dictionary} \times H \text{ encodings}$$

- Sparsity constraints typically imposed on encoding matrix (eg. CNMF-SC)
- Convex Hull NMF: form the dictionary from convex combinations of the points on the convex hull of the input
  - Allows mixed-sign input

$$V \approx S \text{ convex hull points} \times G \text{ convex combinations} \times H \text{ encodings}$$

Convex Hull Convolutive NMF
- Incorporate temporal context in CH-NMF algorithm
- Key advantages over CNMF
  - Relax non-negativity constraint on input
  - Time-varying dictionary has same scale as input

$$V \approx S \text{ convex hull points} \times G(t) \text{ time-varying convex combinations} \times H \text{ encodings}$$

- Find \( K \) temporal patterns with duration \( T \)
- Algorithm:
  1. Input: \( V, K, T, \lambda \) (sparsity level of encoding matrix)
  2. Find convex hull points \( S \) from data in \( V \)
  3. Iteratively update \( G \) and \( H \)

$$G(t) \leftarrow G(t) \odot \left( [S^T V]^+ + [S^T S]^- F \right) H^T, \forall t$$

$$H \leftarrow H - \frac{\sum_{t=0}^{T-1} G(t) \left( [S^T V]^+ - [S^T S]^+ F \right) + \lambda}{\sum_{t=0}^{T-1} G(t) \left( [S^T V]^+ - [S^T S]^+ F \right) + \lambda}$$

4. Return \( S, G(t) \), and \( H \)

Synthetic Data Experiment
- Validate the algorithm
- Create time-series data from 3 Markov chains with 4 states
  - 3 patterns \( \rightarrow K = 3 \)
  - Pattern length of 4 \( \rightarrow T = 4 \)

Articulatory Data Experiment
- Automatically find gestures from vocal tract measurements during speech production

Conclusion & Future Work
- Developed an algorithm to automatically extract temporal patterns from time-series data
- CH-CNMF recovers patterns more faithfully and approximates the input with lower error and higher correlation than CNMF-SC
- Allow \( T \) to vary for different patterns
- Use encoding matrix as features for classification
- Explore joint learning of multiple modalities

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Encodings</th>
<th>RMSE (mm)</th>
<th>Correlation</th>
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</thead>
<tbody>
<tr>
<td>CH-CNMF</td>
<td>( H_{test} )</td>
<td>0.824</td>
<td>0.964</td>
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<tr>
<td></td>
<td>( H_{rand} )</td>
<td>3.419</td>
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<td>CNMF-SC</td>
<td>( H_{test} )</td>
<td>6.058</td>
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<td>( H_{rand} )</td>
<td>8.127</td>
<td>0.168</td>
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</tbody>
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